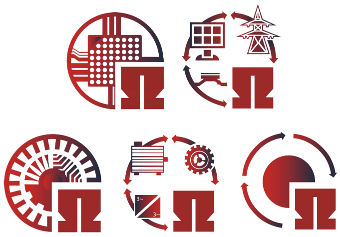
ELSYS Note



Flux Linkage from a 2-D Finite-Element Solution

Flux linkage is a familiar quantity in electrical engineering, whereas the magnetic vector potential used in finite-element analysis is often less intuitive. This note uses Stokes' theorem to establish the connection between both quantities for a planar 2-D problem. A simple three-phase FEMM example then demonstrates how the flux linkage of a distributed winding can be reconstructed from the vector potential and verified using FEMM's circuit properties.

Flux linkage from a finite-element solution

Flux linkage is a familiar quantity in electrical engineering. For a winding with N turns,

$$\psi = N\Phi,$$

where the magnetic flux through a surface S is

$$\Phi = \int_S \mathbf{B} \cdot \mathrm{d}\mathbf{S}. \tag{1}$$

In a finite-element formulation, however, the magnetic flux density $\mathbf{B}$ is generally not the primary solution variable. Instead, the magnetic vector potential $\mathbf{A}$ is introduced through

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{2}$$

For the two-dimensional planar problem considered here, the magnetic vector potential has only a component in the axial direction,

$$\mathbf{A} = A_\mathrm{z}(x, y)\mathbf{i}_\mathrm{z}.$$

Thus, the planar 2-D problem is formulated using the scalar quantity A_z. This simplification does not apply to the general 3-D problem.
Substitution of (2) into (1) gives

$$\Phi = \int_S (\nabla \times \mathbf{A}) \cdot \mathrm{d}\mathbf{S}.$$

Using Stokes' theorem, the surface integral can be transformed into a line integral,

$$\Phi = \oint_C \mathbf{A} \cdot \mathrm{d}\mathbf{l}.$$

Here, C denotes the closed contour bounding the surface S. For a planar model with axial length l_Fe, only the two axial conductor sides contribute to the line integral. For a single turn, this yields

$$\Phi = l_\mathrm{Fe}\left(A_\mathrm{z}^+ - A_\mathrm{z}^-\right).$$

The relation between the familiar flux linkage and the magnetic vector potential used in the finite-element formulation is summarised in Fig. 1.

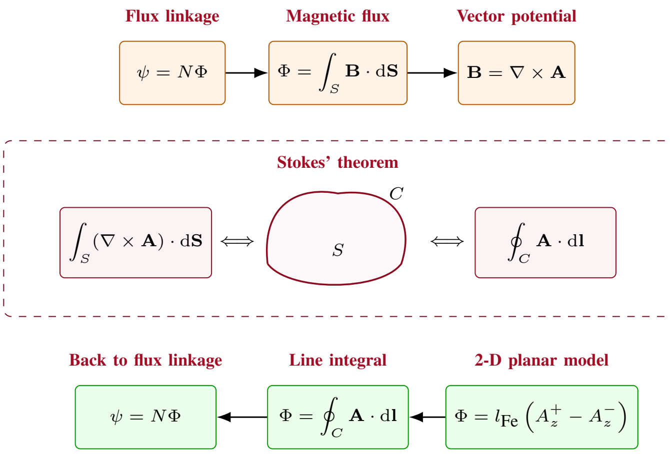


Figure 1. From flux linkage to magnetic vector potential and back. Stokes' theorem links the finite-element solution variable A_z to the familiar electrical-engineering quantity ψ.

Distributed winding

Let k denote the individual coil sides belonging to the considered phase, with $k = 1, \ldots, n_\mathrm{cs}$. In the example considered here, each phase consists of $n_\mathrm{cs} = 6$ coil sides.
A coil side occupies a finite cross-sectional area. The magnetic vector potential associated with coil side k is therefore represented by its average value

$$\overline{A}_{\mathrm{z},k} = \frac{1}{S_k}\int_{S_k} A_\mathrm{z}\,\mathrm{d}S.$$

If N_{jk} denotes the signed number of conductors of phase j in coil side k, the phase flux linkage becomes

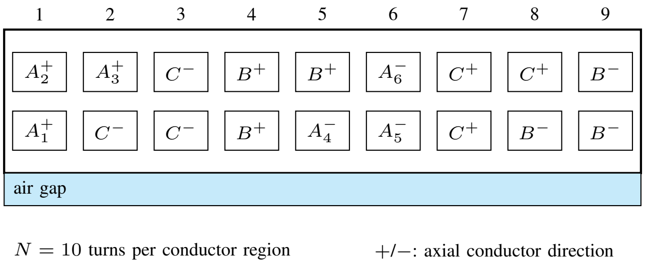
$$\boxed{\psi_j = l_\mathrm{Fe}\sum_{k=1}^{n_\mathrm{cs}} N_{jk}\frac{1}{S_k}\int_{S_k} A_\mathrm{z}\,\mathrm{d}S} \tag{3}$$

The sign of N_{jk} accounts for the coil-side direction. Equation (3) therefore sums the contributions of all coil sides belonging to phase j; corresponding formulations are given in [1, p. 248] and [2, p. 86].

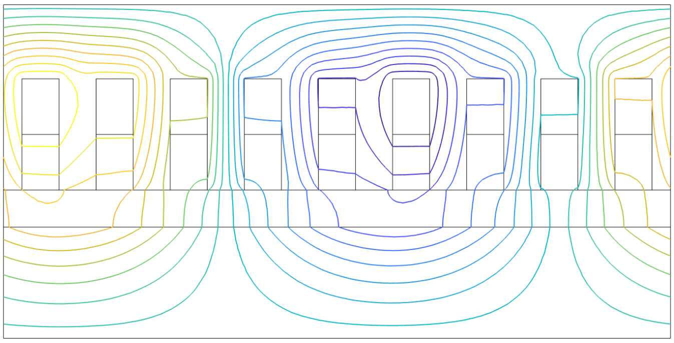
FEMM example

Fig. 2a shows a deliberately simple three-phase winding distribution implemented in FEMM. The conductor regions of each phase are distributed over several slots. The initial current set is

$$i_\mathrm{a} = 10\,\mathrm{A}, \qquad i_\mathrm{b} = i_\mathrm{c} = -5\,\mathrm{A}.$$



(a) Schematic winding arrangement of the FEMM example.



(b) The phase flux linkage is obtained from the mean vector potential within the individual coil-side regions.

Figure 2. FEMM example.

For each conductor region, FEMM provides the integral of the magnetic vector potential and the block cross-sectional area. The contribution of the region can therefore be evaluated by

$$\psi_{jk} = N_{jk} \cdot \frac{\mathrm{Int}_k(A_\mathrm{z})}{S_k}.$$

An important FEMM-specific detail is that, for a planar problem, the quantity returned by the block integral of A_z already contains the specified problem depth. Accordingly,

$$\frac{\mathrm{Int}_k(A_\mathrm{z})}{S_k} = l_\mathrm{Fe}\,\overline{A}_{\mathrm{z},k},$$

and the axial length must not be applied a second time.

Manual and numerical verification

The flux-linkage calculation can first be verified directly in the FEMM postprocessor. After selecting a conductor region, the FEMM block integral provides the integral of the magnetic vector potential and the corresponding value divided by the selected cross-sectional area. For the phase-A_1 coil side, FEMM reports

$$\mathrm{Int}(A_\mathrm{z}) = 4.7868 \times 10^{-13}\,\mathrm{H\,A\,m^2},$$
$$\frac{\mathrm{Int}(A_\mathrm{z})}{S} = 1.276\,48 \times 10^{-6}\,\mathrm{H\,A}.$$

Since $[\psi] = [L\,i] = \mathrm{H\,A} = \mathrm{Wb} = \mathrm{V\,s}$, the latter quantity represents $l_\mathrm{Fe}\,\overline{A}_\mathrm{z}$. With $N = 10$ conductors, the contribution of this coil side to the phase flux linkage is therefore

$$\begin{aligned}\psi_\mathrm{a}(A_1) &= 10 \cdot 1.276\,48 \times 10^{-6} \\ &= 1.276\,48 \times 10^{-5}\,\mathrm{V\,s}.\end{aligned}$$

Repeating this procedure for the six phase-A coil sides gives

$$\sum_{k=1}^{n_\mathrm{cs}=6} \psi_\mathrm{a}(A_k) = 6.777\,442 \times 10^{-5}\,\mathrm{V\,s}.$$

For the negatively oriented coil sides, both N_{Ak} and the corresponding mean value of A_z are negative. Their product therefore gives a positive contribution to the phase flux linkage for the operating point considered here.
The same calculation can be automated using the FEMM–MATLAB postprocessor interface. For each phase-A conductor region, the required quantities are obtained by

```
intA = mo_blockintegral(1);
area = mo_blockintegral(5);

fluxPerTurn = intA/area;
psiA = psiA + N*fluxPerTurn;
```

where N contains the signed number of conductors of the selected coil side.
As an independent check, FEMM provides the flux linkage of a defined circuit directly:

```
B = mo_getcircuitproperties('PhA');
psiA_femm = B(3);
```

For a model depth of $1\,\mathrm{mm}$, the manually reconstructed value, the MATLAB calculation, and the FEMM circuit property all yield

$$\psi_\mathrm{a} = 6.777\,442 \times 10^{-5}\,\mathrm{V\,s}.$$

The relative difference between the MATLAB reconstruction and the FEMM circuit result is approximately $1.399\,757 \times 10^{-15}$ and is therefore limited to numerical precision.
As a further check, increasing the model depth from $1\,\mathrm{mm}$ to $10\,\mathrm{mm}$ increases both results by a factor of ten. Thus, the FEMM block integral already includes the problem depth.

Conclusion

For a planar finite-element model, Stokes' theorem provides the direct link between the solution variable A_z and flux linkage. The FEMM example demonstrates that the phase flux linkage can be reconstructed from the individual coil-side regions and agrees with the circuit property to numerical precision.

References

[1] N. Bianchi, *Electrical Machine Analysis Using Finite Elements*, 1st ed. CRC Press., Jan. 2005, ISBN: 0-8493-3399-7. DOI: 10.1201/9781315219295
[2] S. J. Salon, *Finite Element Analysis of Electrical Machines, Flux linkage*, 1st ed. Springer New York, NY, 1995, Sections 1.3 (p. 10) and 5.4.1 (86). DOI: 10.1007/978-1-4615-2349-9