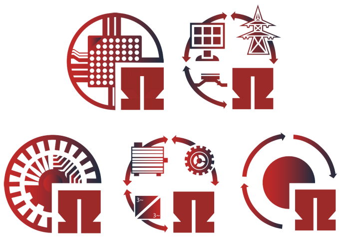
ELSYS Note



From a BH Curve to a FEM Material Model

A BH curve alone is not sufficient for a nonlinear FEM solver. This note revisits a Fortran material routine encountered during my master's research and reconstructs its use of reluctivity, cubic splines, and a smooth high-field continuation. Modern AI tools make such legacy code considerably easier to understand, but interpreting the numerical choices and their relevance to solver convergence remains an engineering task.

A BH Curve is not yet a Material Model

The magnetic properties of electrical steel are commonly described by a BH curve. In commercial finite-element software, providing these data is usually sufficient; interpolation, extrapolation, and the quantities required by the nonlinear solver are handled internally.

For a customised FEM solver, these details become part of the implementation. This was also the case during my master's research on a reluctance synchronous machine, which resulted in the work reported in [1]. The FEM software originated from research on time-stepping analysis of electrical machines [2] and, as is common for research software, had been extended and passed on between several researchers.

One part of the code was a Fortran routine from the early 1990s that converted tabulated BH data into cubic splines, as shown in Fig. 1. Its general purpose was clear at the time, but not the numerical details. Understanding those details requires more than knowing Fortran or spline interpolation: they are closely connected to the formulation and convergence of the nonlinear FEM problem. General FEM references such as [3] provide the theoretical background, but naturally do not explain such solver-specific implementation choices.

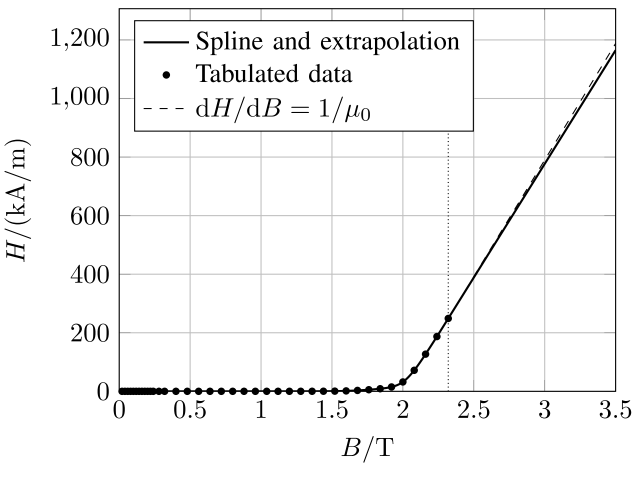


Figure 1. Interpolation of the tabulated BH characteristic and the high-field continuation used by the legacy FEM material model.

From the Field Problem to Reluctivity

For a two-dimensional magnetic field problem, the magnetic vector potential can be chosen as

$$\mathbf{A} = A_\mathrm{z}\,\mathbf{i}_\mathrm{z}.$$

Although only the scalar quantity A_z has to be determined in 2D, its spatial variation describes the magnetic flux density through

$$\mathbf{B} = \nabla \times \mathbf{A}. \tag{1}$$

This is one of the convenient features of the magnetic vector potential: once A_z is known at the nodes of the finite-element mesh, the flux-density components can be obtained from its spatial derivatives.

Starting from Ampère's law, $\nabla \times \mathbf{H} = \mathbf{J}$, and using

$$\mathbf{H} = \nu\mathbf{B}, \qquad \nu = \frac{1}{\mu},$$

together with (1), gives the familiar magnetic field formulation

$$\boxed{\nabla \times (\nu\,\nabla \times \mathbf{A}) = \mathbf{J}}. \tag{2}$$

For air or a linear material, ν is constant. Electrical steel, however, is nonlinear and the reluctivity depends on the local magnetic state. Consequently,

$$\nu = \nu(B^2), \qquad B^2 = \mathbf{B}\cdot\mathbf{B}.$$

The field problem must therefore be solved iteratively. During this process the material model is evaluated locally throughout the mesh and repeatedly during successive nonlinear iterations. Depending on the solution scheme, both the reluctivity and its local gradient are required,

$$\nu(B^2), \qquad \frac{d\nu}{d(B^2)}. \tag{3}$$

This also explains a numerical detail of the original solver. Working directly with B^2 avoids repeatedly calculating

$$B = \sqrt{B_\mathrm{x}^2 + B_\mathrm{y}^2}.$$

A single square root is insignificant, but material evaluation is performed for many elements and repeated during every

nonlinear iteration. For software developed in the early 1990s, such computational savings were worth considering.
The BH data supplied to the program must therefore be converted into a form that provides the quantities in (3) efficiently and smoothly. This is the purpose of the spline routine.

The Numerical Material Model

The routine does not interpolate $H(B)$ directly. Instead, the input data (B_i, H_i) are transformed to

$$x_i = B_i^2, \qquad \nu_i = \frac{H_i}{B_i},$$

and cubic splines are constructed for $\nu(B^2)$, as shown in Fig. 2. This is convenient for the finite-element formulation and provides the two quantities required repeatedly during the nonlinear solution,

$$\nu(B^2), \qquad \frac{d\nu}{d(B^2)}.$$

Using $B^2 = \mathbf{B} \cdot \mathbf{B}$ also avoids repeatedly evaluating the magnitude of $\mathbf{B}$. Such computational details were particularly relevant when the solver was developed.

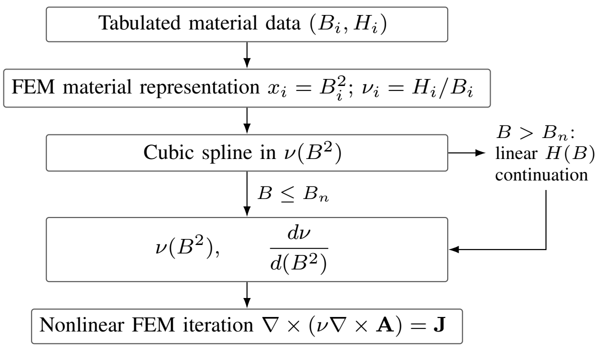


Figure 2. From tabulated BH data to the quantities required by the nonlinear FEM solver. Beyond the final data point, the original implementation switches to a linear BH continuation.

The more interesting question is what happens when an iteration produces a flux density beyond the final tabulated point. The material model cannot simply end there. Physically, the differential permeability should eventually approach μ_0, suggesting a linear continuation with

$$\frac{dH}{dB} \to \frac{1}{\mu_0}.$$

The original implementation, however, uses the apparently nonlinear extrapolation

$$\nu(B) = \nu_n + 2g_n B_n^2 \left(1 - \frac{B_n}{B}\right), \qquad g_n = \left.\frac{d\nu}{d(B^2)}\right|_{B_n}.$$

Multiplication by B reveals the underlying idea:

$$\boxed{H(B) = H_n + \left(\nu_n + 2g_n B_n^2\right)(B - B_n)}.$$

Thus, beyond the final spline point the BH characteristic becomes *exactly linear*, while its reluctivity and reluctivity gradient remain continuous at the transition.

For the example in Fig. 1, with the final data point at $B_n = 2.32\,\mathrm{T}$, the resulting slope is

$$\frac{dH}{dB} = 7.763 \times 10^5 \,\frac{\mathrm{A/m}}{\mathrm{T}} = 0.9755\,\frac{1}{\mu_0}.$$

The continuation therefore approaches the expected air-line behaviour without forcing an abrupt change of the quantities used by the nonlinear solver. For an electrical machine, flux densities beyond this range are normally of little interest for the final solution, but they may occur temporarily during iteration. A smooth and well-defined continuation can therefore be more valuable numerically than an exact description of the steel at extreme flux densities.

Revisiting Legacy Engineering Software

During my master's work, the spline routine was part of an inherited FEM solver. Its purpose was understood, but the research question concerned the electrical machine rather than the numerical details of the software. Revisiting the same code many years later shows how much has changed: tools such as ChatGPT allow legacy code and its underlying mathematics to be reconstructed in a fraction of the time. Engineering knowledge, however, remains essential to interpret why a particular numerical approach was chosen.

Conclusion

Over the years, commercial FEM tools have largely removed the need to develop customised field solvers for many engineering applications. This alone represents a major change from the environment in which the software discussed here was developed. A second change is now taking place. Tools such as ChatGPT make it possible to understand and restructure complex legacy code in a remarkably short time. A routine that remained largely a black box during my master's research could now be traced back to its mathematical and numerical purpose within hours. In both cases, however, the engineer remains essential. Commercial software can hide the numerical implementation, and AI can accelerate its reconstruction, but recognising why a particular numerical treatment is useful, such as the smooth transition into the high-field region, still requires an understanding of the underlying physics and numerical method.

References

[1] J.J. Germishuizen, F. v. der Merwe, K. v. der Westhuizen, and M.J. Kamper, "Performance comparison of reluctance synchronous and induction machines traction drives for electrical multiple units," *IEEE IAS*, vol. 1, Oct. 2000, pp. 316–323. DOI: 10.1109/IAS.2000.881130

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[3] P. P. Silvester and R. R. Ferrari, *Finite elements for electrical engineers*. Cambridge University Press, 1996. DOI: 10.1017/CBO9781139170611 [Online]. Available: http : / / www . cambridge . org / gb / knowledge / isbn / item1143972/?site_locale=en_GB