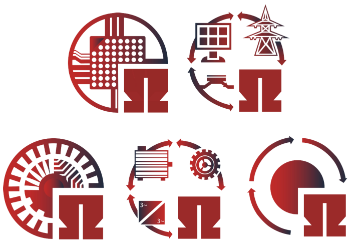
ELSYS Note



Three halves

*The factor 3/2 appears frequently in three-phase electrical systems, yet its origin is often not explicitly discussed. Starting from a single concentrated winding (**C**), this note illustrates how distributed windings (**D**) and balanced three-phase excitation lead to a rotating fundamental field. The resulting equivalent single-phase representation reveals that the fundamental amplitude increases by a factor of 3/2, explaining its recurring appearance.*

Three halves

Engineers working with electrical systems are familiar with a small set of constants: $\sqrt{2}$, $\sqrt{3}$, and π. Frequency values such as 50 Hz [1, 2], and even 16.7 Hz and 16.667 Hz in European railway systems have well-known historical and technical origins.
But there is another number that repeatedly appears in three-phase systems, often without much discussion: the factor 3/2.
Where does it come from?

From one slot to three

We start with a single concentrated stator winding. The resulting air-gap flux density shows a pronounced fundamental component, but also significant harmonic content. This is shown in Fig. 1. In this example the air-gap is $\delta = 0.75\,\text{mm}$ resulting in an air-gap flux density of $B_\text{r} = 0.123\,\text{T}$.
By distributing the winding over several slots, the harmonic content is reduced and the fundamental becomes more dominant, while the total magnetomotive force (mmf) remains unchanged. This is shown in Fig. 2. The THD reduces from 47 % to 26.3 % and the reduction of the working harmonic is due to the winding factor k_w. It is important to notice that the total mmf in both cases is $\Theta = 150\,\text{A}$. Consequently, the $B_{\text{r,max}}$ component is unchanged.

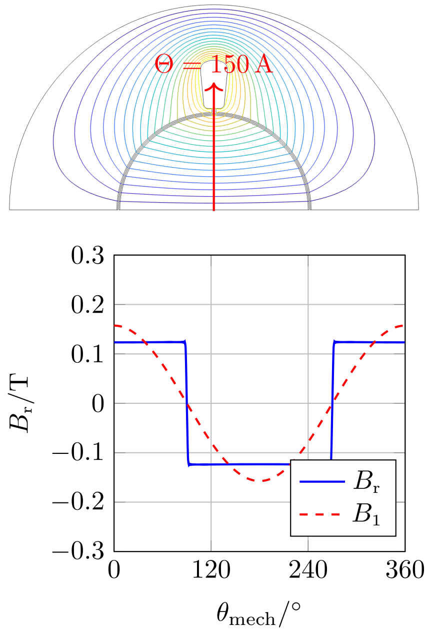


Fig. 1: Concentrated winding

Adding the other two phases

Extending this concept to a three-phase system, two additional phase windings are introduced, each displaced by 120° in space and excited by currents shifted by 120° in time. Although each phase produces the same peak mmf distribution, their spatial and temporal displacement leads to a constructive combination of the fundamental component.
As shown in Fig. 3(a), the total mmf of the distributed three-phase system is $\sum \Theta = 300\,\text{A}$, while the harmonic content is significantly reduced. The distributed winding ensures little harmonic content as there is a good correlation between the total and working harmonic. Again the reduction of the working harmonic is due to the winding factor k_w.

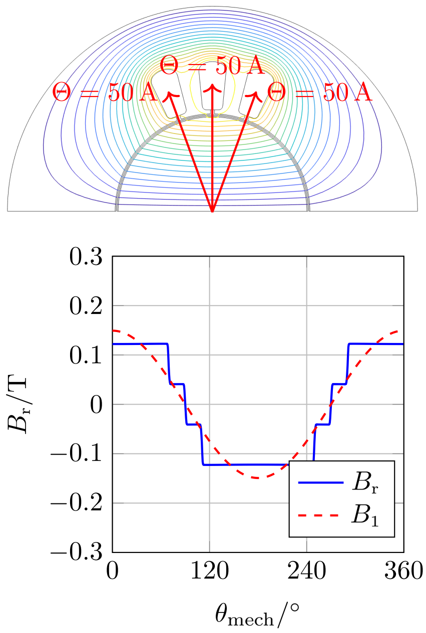
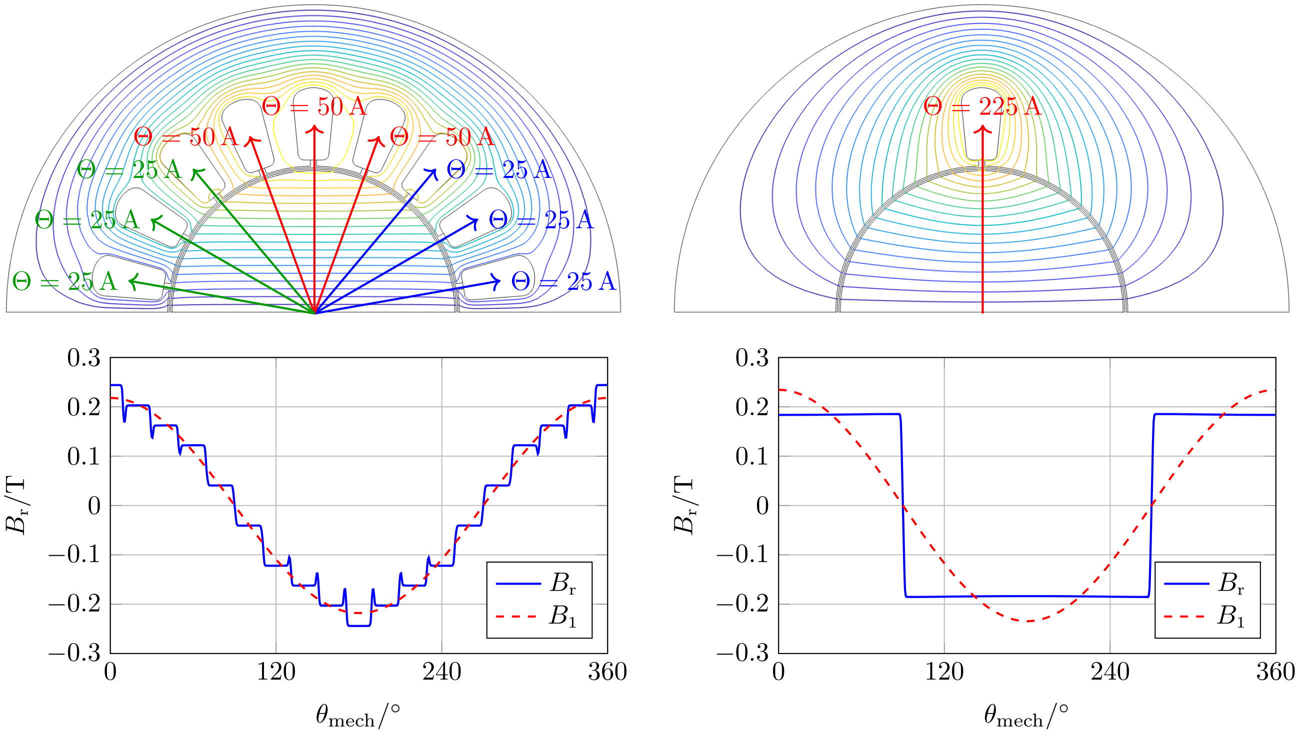


Fig. 2: Distributed winding



(a) Distributed three-phase winding: $\sum \Theta = 300\,\mathrm{A}$ (b) Three-phase per phase equivalent: $\sum \Theta = 225\,\mathrm{A}$

Fig. 3: Definition of the fundamental wave

A brief summary of the properties are given in the table below.

Θ/A	B_r/T	B_1/T	THD/%
150-**C**	0.123	0.156	47.2
150-D	0.122	0.149	26.3
300-C	0.245	0.232	30.4
300-D	0.244	0.218	12.6
225-**C**	0.184	0.234	47.2

The three-phase concentrated winding has a working harmonic amplitude of $B_1 = 0.232\,\mathrm{T}$.

Back to one phase

For analysis, it is common to replace the three-phase system by an equivalent single-phase representation. This equivalent system must reproduce the same fundamental air-gap field. As shown in Fig. 3(b), this is achieved with an equivalent mmf of $\Theta = 225\,\mathrm{A}$, resulting from the phasor addition of the three phase contributions in Fig. 3(a). Including the winding factor yields the air-gap flux density of the distributed three-phase winding.

The origin of three halves

Comparing the single-phase case with the equivalent representation reveals

$$225 = \frac{3}{2} \cdot 150.$$

The factor 3/2 appears when three spatially and temporally shifted phase contributions combine into a single rotating fundamental field whose amplitude is 3/2 of a single phase contribution. This equivalent representation corresponds to a fundamental wave producing the same air-gap field. It is therefore not a correction factor, but a direct consequence of the geometry of balanced three-phase systems.

Final remark

The same factor reappears in many areas of electrical engineering, including power expressions and transformed machine models. Whether it appears explicitly depends on the chosen normalisation, but its origin remains the same: the superposition of three phases.

References

[1] Eric Laithwaite. “Why do we use 3 phase 50 Hz?” *Electrical review* 212.5 (1983), p. 26.

[2] Gerhard Neidhöfer. “Der Weg zur Normfrequenz 50 Hz”. *Bulletin SEV/AES* 17 (2008), pp. 29–34.