ELSYS Note

The Harmonic Tonic

This ELSYS Note investigates harmonic analysis in the context of spatial distributions, particularly in electrical machines. It highlights the differences between spatial and time-domain analysis and explains the significance of the working harmonic. Using rectangular functions, it demonstrates how harmonic analysis is performed in both domains and how this enables deeper insights into complex systems.

Introduction

Harmonic analysis is a powerful tool for investigating signals and distributions that play an important role in many technical applications. In this ELSYS Note, we take a closer look at harmonic analysis, with particular emphasis on the considerations that arise when it is applied to spatial distributions.

Background

Harmonic analysis in the time domain is familiar to many and is commonly used to study higher-order harmonics above the working harmonic and subharmonics below the working harmonic in time-dependent signals. This method has proven to be extremely useful. In many areas of engineering, harmonic analysis has become an integral part of everyday professional practice.

Spatial Domain

In electrical machines, spatial distributions often need to be analysed, for example, the magnetic flux density in the air-gap. In this context, the radial component of the flux density is typically evaluated as a function of the circumferential position or pole pitch.

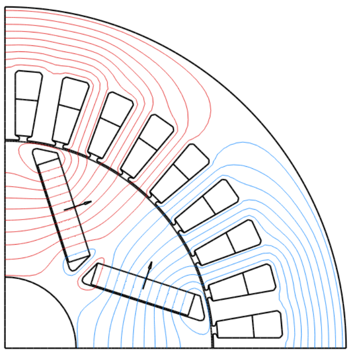


Fig. 1: V-shaped PMSM

Fig. 1 shows a permanent magnet synchronous machine with V-shaped magnets. The machine geometry is described in detail in [1]. This example raises the question of how harmonic analysis can be applied to purely spatial functions. A key requirement is the distinction between the fundamental and the working harmonic. An algebraic approach to determine machine harmonics is given in [2].

Working Harmonic

The magnetic field distribution in the air-gap of an electrical machine is governed by several interacting factors, including winding layout, rotor geometry, and excitation [2]. According to the definition by Heller and Hamata [3], all spatial harmonics are referenced to the full mechanical circumference of the machine.

The fundamental harmonic of order $\nu = 1$ corresponds to a single pole pair and spans a circumferential angle of 2π. If the machine has p pole pairs, the harmonic that produces a magnetic field with exactly p pole pairs is termed the *working harmonic*. A simplified stator representation illustrating this concept is shown in Fig. 2.

The smallest possible spatial wave has a wavelength of 2π and is referred to as the fundamental harmonic. Consequently, the working harmonic is always of order $\nu = p$. In general, a harmonic of order n generates n pole pairs, with each pole pair occupying a circumferential angle of $2\pi/n$.

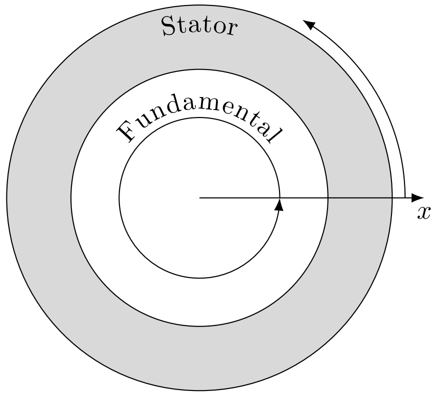
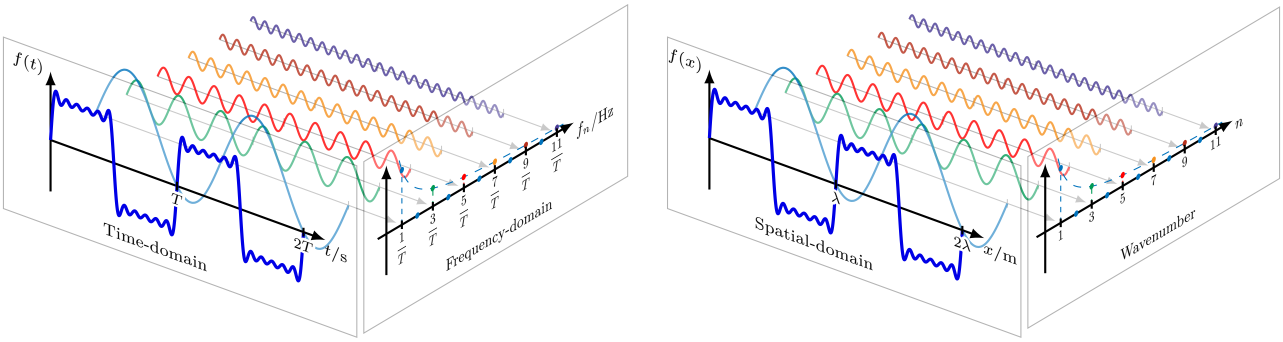


Fig. 2: Fundamental



(a) Time-domain

(b) Spatial-domain

Fig. 3: Harmonic analysis in the time and spatial domains

Rectangular Signal

To explain the basic principle, we consider a well-known rectangular signal up to the 11th harmonic. This can be expressed as a Fourier series:

$$f(t) = \frac{4}{\pi} \sum_{1,3,\ldots}^{11} \frac{1}{n} \sin\left(\frac{2\pi}{T} nt\right)$$

The fundamental component and the 11th harmonic have periods of T and $T/11$, respectively. According to the sampling theorem, the time signal must be sampled with a sampling interval smaller than $T/(2 \cdot 11)$ to ensure that aliasing does not occur.

The harmonic analysis is illustrated in Fig. 3a. In the time domain, the rectangular signal can be observed, while in the frequency domain the individual frequencies (harmonics) are clearly identified. Through harmonic analysis, the time signal is decomposed into its individual components. In many engineering applications, knowledge of these components is essential, as they provide valuable insight into system behavior.

Rectangular Wave

To explain the principle of spatial harmonic analysis, the same rectangular wave is represented in the spatial domain:

$$f(x) = \frac{4}{\pi} \sum_{1,3,\ldots}^{11} \frac{1}{n} \sin\left(\frac{2\pi}{\lambda} nx\right)$$

This expression is identical, except for the labeling of the x-axis. Instead of time, we now consider distance (in m) or alternatively an angular coordinate (in rad). The rectangular wave has a working harmonic and an 11th harmonic with wavelengths of λ and $\lambda/11$. The harmonic analysis is shown in Fig. 3b.

The sampling theorem states that sampling should be performed with a spatial resolution of at least $\lambda/(2 \cdot 11)$.

In the harmonic domain, when considering spatial representations, the unit is typically the spatial harmonic number or simply wave number (n). The unit of the spatial wave number is usually $1/\mathrm{m}$ or $1/\mathrm{rad}$, depending on the chosen spatial unit (e.g., meters or radians). The spatial wave number is the counterpart to frequency in the time domain and indicates how many wave cycles exist per unit length.

Outlook

Spatial harmonic analysis is a valuable method for understanding complex systems. By connecting spatial- and time-domain perspectives, it enables deeper insight and supports the development of innovative solutions.

References

[1] J.J. Germishuizen and T.J.E. Miller. “Inductance Analysis of Electric Machines by Classical and Numerical Methods”. *2022 24th European Conference on Power Electronics and Applications (EPE’22 ECCE Europe)*. IEEE Xplore. 2022, pp. 1–9.

[2] Johannes Germishuizen and Andreas Kremser. “Algebraic Design of Symmetrical Windings for AC Machines”. *IEEE Transactions on Industry Applications* 57.3 (2021), pp. 1928–1934. DOI: 10.1109/TIA.2021.3057607.

[3] B. Heller and V. Hamata. *Harmonic Field Effects in Induction Machines*. Czechoslovak Academy of Sciences. Elsevier Scientific Publishing Company, 1977. ISBN: 9780444998569.