# **ELSYS Note**



Technische Hochschule

# The Secrets of Hybrid Electric Machines

This note examines hybrid electric machines, focusing on the Reluctance-Assisted PMSM. By combining permanent magnet and reluctance torque, these machines achieve superior field weakening and torque performance, making them ideal for applications like electric vehicles.

#### Introduction

Synchronous machines are widely used in modern applications, especially in electromobility. Among them, the Permanent Magnet Synchronous Machine (PMSM) is one of the most common. For applications that require extended field weakening, a hybrid type called the Reluctance-Assisted PMSM offers significant advantages. This note explains the principles of this machine. To provide a clear understanding, we first examine the basic operation of the Synchronous Reluctance Machine (SynRM) and the PMSM separately. We then combine these concepts to highlight the motivation and benefits of hybrid designs.

## The Synchronous Reluctance Machine

The SynRM (Fig. 2(a)) operates based on the anisotropic nature of its rotor, which is made entirely of laminated materials. For simplicity, we consider a two-pole machine (p = 1), as illustrated in Fig. 1, where the electrical and mechanical angles are equal. Torque in this machine arises from the tendency of the rotor *d*-axis to align with the magnetic flux in the machine. Equivalently, the rotor *q*-axis tends to align with the space current phasor, which in Fig. 1 lags by the current angle  $\theta$ .

Electromagnetically, this torque results from the cross-product of the stator space current and the flux linkages, i.e.  $T \propto \psi \times \mathbf{i}$ . Mathematically, it is expressed as:

$$\begin{split} \vec{T} &= \frac{3}{2} p \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \psi_{\mathrm{x}} & \psi_{\mathrm{y}} & \psi_{\mathrm{z}} \\ i_{\mathrm{x}} & i_{\mathrm{y}} & i_{\mathrm{z}} \end{vmatrix} \\ T_{\mathrm{z}} &= \frac{3}{2} p \left[ (\psi_{\mathrm{y}} i_{\mathrm{z}} - \psi_{\mathrm{z}} i_{\mathrm{y}}) \hat{i} \\ &+ (\psi_{\mathrm{z}} i_{\mathrm{x}} - \psi_{\mathrm{x}} i_{\mathrm{z}}) \hat{j} \\ &+ (\psi_{\mathrm{x}} i_{\mathrm{y}} - \psi_{\mathrm{y}} i_{\mathrm{x}}) \hat{k} \right]. \end{split}$$

Since the space phasors lie in the xy-plane, only the z-component contributes to the torque, which is generated about the z-axis. In machine terms, the x- and y-axes correspond to the rotor's d- and q-axes, while the z-axis represents the rotor shaft. Substituting these axis definitions, the torque simplifies to:

$$T = \frac{3}{2} p \left( \psi_{\mathrm{d}} i_{\mathrm{q}} - \psi_{\mathrm{q}} i_{\mathrm{d}} \right)$$
$$= \frac{3}{2} p \left( L_{\mathrm{d}} - L_{\mathrm{q}} \right) i_{\mathrm{d}} i_{\mathrm{q}}.$$

Here,  $L_{\rm d}$  and  $L_{\rm q}$  are the inductances along the *d*- and *q*-axes, respectively, and  $i_{\rm d}$  and  $i_{\rm q}$  are the currents in these axes. The torque is directly proportional to the inductance difference,  $L_{\rm d} - L_{\rm q}$ . To maximize torque for a given stator current, machine design aims to maximize this difference. However, mechanical constraints limit the extent to which this optimization can be achieved.



# Enhancing Torque with

**Permanent Magnets** 

equation as follows:

To further increase the torque capability,  $L_q$  can be reduced by embedding permanent magnets along the q-axis. These magnets create a flux that opposes the linkage in the q-axis, which modifies the torque

$$T = \frac{3}{2} p \left( \psi_{\rm d} i_{\rm q} - (\psi_{\rm q} - \psi_{\rm PM}) i_{\rm d} \right)$$
  
=  $\frac{3}{2} p \left( \psi_{\rm PM} i_{\rm d} + (L_{\rm d} - L_{\rm q}) i_{\rm d} i_{\rm q} \right)$ 





This simplified equation shows that

the torque in a pure PMSM depends

entirely on the permanent magnet

This equation shows that the torque now comprises two components:

- Reluctance torque: Proportional to  $(L_{\rm d} L_{\rm q})i_{\rm d}i_{\rm q}$ , arising from the inductance difference between the *d* and *q*-axes.
- Magnetic torque: Proportional to  $\psi_{\rm PM} i_{\rm d}$ , generated by the embedded permanent magnets.

This hybrid design not only enhances torque production but also significantly improves field weakening capabilities, providing greater flexibility in machine operation.

### The Pure PMSM

For comparison, let us examine the pure *Permanent Magnet Synchronous Machine* (PMSM). As before, we consider a two-pole machine for simplicity (Fig. 2(d)). The general torque equation is given by:

$$T = \frac{3}{2} p \, \left( \psi_{\rm PM} i_{\rm q} + \left( L_{\rm d} - L_{\rm q} \right) i_{\rm d} i_{\rm q} \right)$$

In a pure PMSM, which typically employs a surface-mounted rotor design, the inductances  $L_d$  and  $L_q$ are equal. This is because the relative permeability of the permanent magnets is approximately 1, and the rotor exhibits no anisotropy. As a result, the torque equation simplifies to:

$$T = \frac{3}{2} p \psi_{\rm PM} i_{\rm q}.$$

Reluctance Effects
The provide the proton, the proton is added to the rotor, the proton is added to the rotor.

flux linkage  $\psi_{\rm PM}$  and  $i_{\rm q}$ .

inductances  $L_{\rm d}$  and  $L_{\rm q}$  become unequal, with  $L_{\rm q} > L_{\rm d}$ . Substituting into the torque equation, the term  $(L_{\rm d} - L_{\rm q})i_{\rm d}$  introduces a negative torque component, reducing the total torque. To ensure proper operation, the  $i_{\rm d}$ -current must therefore be negative.

Interestingly, in this configuration,  $L_{\rm d}$  is smaller than  $L_{\rm q}$ . This is the opposite of what is observed in a pure reluctance machine. A comparison between Fig. 2(a) and Fig. 2(c) reveals an important distinction: the "**pure SynRM**" and the "**pure PMSM**" with reluctance effects are oriented perpendicularly. The *d*-axis of the SynRM in Fig. 2(c) aligns with the *q*-axis of the PMSM with reluctance.

### **Field Weakening**

In the field weakening range, the terminal voltage must remain below the power supply's maximum output,  $U \leq U_{\text{max}}$ , as illustrated in Fig. 3. Typically, the point where maximum voltage is reached is  $n_1$ . To achieve this, a field weakening current,  $i_{\text{d}} < 0$ , is introduced. Since

 $L_{\rm d} - L_{\rm q} < 0$ , the term  $(L_{\rm d} - L_{\rm q})i_{\rm d}$ becomes positive, adding a reluctance torque component.



Fig. 3: Field weakening

In this range,  $i_{\rm q}$ -current, and thus the torque contribution from the permanent magnets, is reduced. However, the negative  $i_{\rm d}$ -current controls the terminal voltage and enhances torque via the reluctance effect, reducing the total stator current. This improvement is particularly beneficial for designs with weaker magnets.

Reluctance torque is primarily advantageous in applications requiring field weakening. In cases where field weakening is unnecessary, adding reluctance features is redundant.

### Conclusion

The interplay of permanent magnet and reluctance torque in PMSMs highlights the advantages of hybrid designs. By managing currents and leveraging rotor anisotropy, these machines deliver superior performance in applications demanding field weakening, such as electric vehicles.