

Observer-Controller Design and Simulation of a High-Speed MAGLEV Vehicle

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Abstract

A magnetically levitated vehicle (MAGLEV) is simplified by a single mass system on a rigid guideway. The equations of motion of the electromechanical components are derived with LAGRANGE-function in state space notation. Because of the instability of the electromagnetic levitation system the vehicle must be actively controlled. The controller is designed by Linear Quadratic Control. An observer (KALMAN-Filter) is used to estimate the states which can not be measured or are influenced by noise. For the simulation of the controlled system the toolbox MATLAB is used. Simulation results for the time histories of the magnet gap and the acceleration are given for a gap initial condition.

The research work was supported by the Education Committee and Science and Technology Committee of Shanghai Municipality, P.R.China. It was carried out during the time Prof. Dr. Reinhold Meisinger was visiting professor in Shanghai 2008 and Prof. Shu Guangwei, M.Sc. was visiting professor in Nuremberg 2009.

1. Introduction

The MAGLEV train is one of the most important technology innovations in Germany in the end of the last century. It is the first train system which operates completely non-contact and without wheels [1, 7]. Because of the instability of the electro-magnetic levitation system the electromagnets must be actively controlled. The equations of motion are derived with LAGRANGE-function for a simplified single mass vehicle in state space notation [2, 5]. In engineering practice the vertical velocity of the vehicle cannot be measured by a sensor and the measurement of gap and acceleration are influenced by noise. So an observer is used to estimate it [2]. For the control system design a quadratic criterion is used. There the magnet gap, the vehicle vertical velocity and acceleration are weighted (Linear Quadratic Control) [2, 3]. For the observer design it is assumed that the system input and the measurements are influenced by white GAUSSIAN noises (KALMAN-Filter). The computer simulation of the active system is carried out with the toolbox MATLAB [6]. Simulation results of the direct state feedback and the use of an observer controller for the time histories of the magnet gap and the acceleration are given for a gap initial condition.

2. Technical Model

In Fig.1 the cross-section of the technical model of a MAGLEV vehicle without secondary suspension is shown. The electromagnets which generate attractive forces are fixed rigidly under the cabin. Because the vehicle is unstable, the magnet gap s_M and the vertical acceleration \ddot{s}_M are continuously measured by sensors and used as an input in the controller. The controller is computing the magnet voltage u_M so that the vehicle is levitating. Note: The vertical velocity which is necessary for the control is estimated by an observer [1, 2].

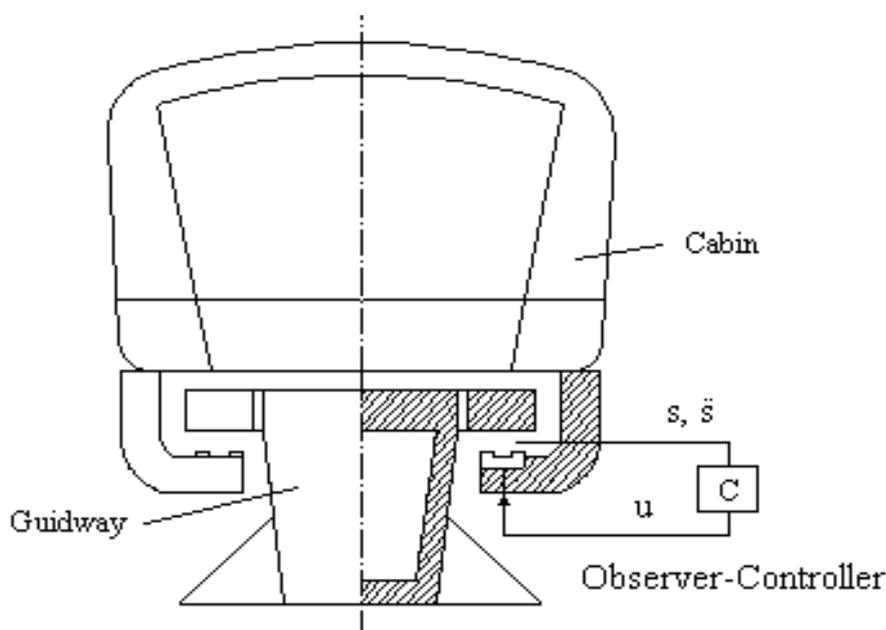


Fig. 1: Technical model of MAGLEV vehicle

3. Mathematical Model

In Fig.2 the mathematical model of a single mass vehicle on a rigid guideway is shown. There mg is the vehicle weight (including the magnets), s_M is the magnet gap, R is the resistance, $L(s_M)$ is the gap dependent inductance, i_M is the magnet current and u_M is the magnet voltage. Stray flux as considered in [2] and eddy current effects will be neglected.

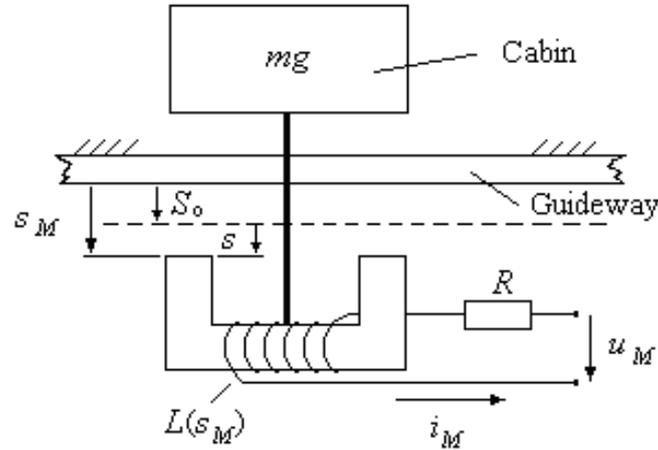


Fig. 2: Mathematical model of a single mass MAGLEV vehicle

3.1 Nonlinear differential equations

The equations of motion can be derived e.g. with LAGRANGE-function L , which is defined as difference between kinetic energy T and potential energy U of a dynamic system.

$$L = T - V \quad (1)$$

Then the LAGRANGE equations of motion are [3]

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_r} \right) - \frac{\partial L}{\partial q_r} = Q_r, \quad r = 1(1), \quad (2)$$

where q_r are the generalized coordinates and \dot{q}_r are the generalized velocities who describe the system in the n -dimensional vector-space. Q_r are the generalized forces who cannot derived from a potential.

With the relative permeability of iron $\mu_r \gg 1$ and the inductance constant μ_0 the inductance $L(s_M)$ of the magnet coil can be written as [4]

$$L(s_M) = \frac{\mu_0 \cdot N^2 \cdot A}{2 \cdot s_M}$$

where A is the pole area and N is the number of windings.

With the generalized coordinate $q_1 = s_M$ and the generalized velocities $\dot{q}_1 = \dot{s}_M$, $\dot{q}_2 = i_M$ the kinetic energy T and the potential Energy U of the MAGLEV system are

$$T = \frac{1}{2} L(s_M) \cdot i_M^2 + \frac{1}{2} m \cdot \dot{s}_M^2, \quad (3)$$

$$U = -m \cdot g \cdot s_M.$$

Then the LAGRANGE-function is obtained according eq. (1) to

$$L = \frac{1}{2} L(s_M) \cdot i_M^2 + \frac{1}{2} m \cdot \dot{s}_M^2 + m \cdot g \cdot s_M. \quad (4)$$

The generalized force in eq. (2) is $Q_2 = u_M - i_M \cdot R$.

With eq. (2) and eq. (4) the nonlinear equations of motion are obtained as:

$$\ddot{s}_M = -\frac{L(s_M)}{2 \cdot s_M \cdot m} i_M^2 + g, \quad (5)$$

$$\dot{i}_M = -\frac{R}{L(s_M)} i_M + \frac{i_M}{s_M} \dot{s}_M + \frac{1}{L(s_M)} u_M.$$

3.2 Linear differential equations

The linearization is useful for the design of the Linear Quadratic Control (RICCATI-controller) and the design of the observer (KALMAN-Filter). For small deviations from steady-state we can write, cf. Fig.2

$$s_M = S_0 + s, \quad i_M = I_0 + i, \quad u_M = U_0 + u, \quad \dot{s}_0 = 0, \quad \dot{i}_0 = 0.$$

Together with eq. (5) and the values in steady-state position

$$L_0 = L(S_0), \quad U_0 = I_0 \cdot R,$$

the following linearized equations of motion are obtained:

$$\begin{aligned} \ddot{s} &= -\frac{P_i}{m} i + \frac{P_s}{m} s, \\ i &= -\frac{R}{L_0} i + \frac{I_0}{S_0} \dot{s} + \frac{1}{L_0} u, \end{aligned} \quad (6)$$

with the magnet parameters

$$P_i = \frac{L_0 \cdot I_0}{m \cdot S_0}, \quad P_s = \frac{P_i \cdot I_0}{S_0}.$$

Eq. (6) can be transformed into

$$\ddot{s} = \frac{P_s R}{m L_0} s - \frac{R}{L_0} \dot{s} - \frac{P_i}{m L_0} u \quad (7)$$

4. State Space Notation

Together with eq. (7) and s, \dot{s} as measurements the following state equation and measurement equation are obtained [2]:

$$\begin{aligned} \frac{d}{dt} \begin{bmatrix} s \\ \dot{s} \\ \ddot{s} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ P_s R / (m L_0) & 0 & -R / L_0 \end{bmatrix} \cdot \begin{bmatrix} s \\ \dot{s} \\ \ddot{s} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -P_i / (m L_0) \end{bmatrix} \cdot u \\ \begin{bmatrix} s \\ \dot{s} \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s \\ \dot{s} \\ \ddot{s} \end{bmatrix} \end{aligned} \quad (8)$$

Eq. (8) has the form

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A} \cdot \mathbf{x} + \mathbf{D} \cdot u \\ \mathbf{y} &= \mathbf{C} \cdot \mathbf{x} \end{aligned}$$

with the state vector \mathbf{x} , the scalar control input signal u , the measurement vector \mathbf{y} , the system matrix \mathbf{A} , the input matrix \mathbf{D} , and the measurement matrix \mathbf{C} .

The control law for state feedback is

$$u = -\begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \cdot \begin{bmatrix} s \\ \dot{s} \\ \ddot{s} \end{bmatrix}. \quad (9)$$

With the controller feedback matrix \mathbf{K} eq. (9) has the form

$$u = -\mathbf{K} \cdot \mathbf{x}$$

5. Controller Design

To optimize the system the quadratic loss criterion

$$J = \frac{1}{2} \int_0^{\infty} [\mathbf{x}^T \cdot \mathbf{C}^T \cdot \mathbf{Q} \cdot \mathbf{C} \cdot \mathbf{x} + R \cdot u^2] \cdot dt = \min \quad (10)$$

is used. There the diagonal matrix

$$\mathbf{Q} = \text{diag}[\mathbf{Q}_1 \quad \mathbf{Q}_2]$$

is weighting with \mathbf{Q}_1 the gap (safety) and with \mathbf{Q}_2 the acceleration (ride comfort). The scalar quantity R is weighting the input signal (control power) [2].

Then the controller feedback matrix \mathbf{K} is obtained as

$$\mathbf{K} = \frac{1}{R} \mathbf{D}^T \cdot \mathbf{P}, \quad (11)$$

where \mathbf{P} is the steady-state solution of the matrix RICCATI equation

$$\mathbf{P} \cdot \mathbf{A} + \mathbf{A}^T \cdot \mathbf{P} + \mathbf{C}^T \cdot \mathbf{Q} \cdot \mathbf{C} - \mathbf{P} \cdot \mathbf{D} \cdot \mathbf{K} = \mathbf{0}. \quad (12)$$

This equation can be solved e.g. numerically with MATLAB [6].

6. Observer Design

Because in practice the velocity \dot{s} can not be measured by a sensor and the gap and acceleration measurements s, \ddot{s} are influenced by noise an estimation of the state vector \mathbf{x} is performed by the following 3rd order observer (KALMAN-Filter):

$$\dot{\hat{\mathbf{x}}} = \mathbf{A} \cdot \hat{\mathbf{x}} + \mathbf{D} \cdot \mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C} \cdot \hat{\mathbf{x}}) \quad (13)$$

Eq. (13) is a model of the system eq. (8) and a feedback signal proportional to the difference between the actual measurement signal \mathbf{y} and the estimated measurement signal $\mathbf{C} \cdot \hat{\mathbf{x}}$. It is assumed that the system input and the measurements are influenced by uncorrelated stochastic vector processes which are white GAUSSIAN noises

$$\mathbf{w} \sim N[0, \mathbf{G}], \quad \mathbf{v} \sim N[0, \mathbf{H}] \quad (14)$$

with mean value zero. The scalar quantity G is the power spectral density of the system input noise and the diagonal matrix

$$\mathbf{H} = \text{diag}[H_1, H_2]$$

describes with H_1, H_2 the power spectral densities of the gap and acceleration measurement noises.

Then the observer feedback matrix \mathbf{L} is obtained as

$$\mathbf{L} = \mathbf{\Sigma} \cdot \mathbf{C}^T \cdot \mathbf{H}^{-1} \quad (15)$$

where $\mathbf{\Sigma}$ is the steady-state solution of the matrix RICCATI equation

$$\mathbf{A} \cdot \mathbf{\Sigma} + \mathbf{\Sigma} \cdot \mathbf{A}^T + \mathbf{D} \cdot \mathbf{G} \cdot \mathbf{D}^T - \mathbf{L} \cdot \mathbf{C} \cdot \mathbf{\Sigma} = \mathbf{0}. \quad (16)$$

Because of the duality between Linear Quadratic Control and KALMAN-Filter, eq. (16) can be solved numerically similar to eq. (12) when the relations shown in table 1 are used.

Linear Quadratic Control	Observer (KALMAN-Filter)
A	A^T
D	C^T
Q	G
R	H
K	L^T
P	Σ^T

Table 1: Duality between controller and observer

7. Simulation with MATLAB

The system eq. (8) and the observer eq. (13) can be combined with the control law

$$u = -K \cdot \hat{x} \tag{17}$$

to the following matrix differential equation used for the simulation with MATLAB:

$$\frac{d}{dt} \begin{bmatrix} x \\ \hat{x} \end{bmatrix} = \begin{bmatrix} A & -DK \\ LC & A-DK-LC \end{bmatrix} \cdot \begin{bmatrix} x \\ \hat{x} \end{bmatrix} \tag{18}$$

The block diagram shown in Fig. 3 demonstrates eq. (18).

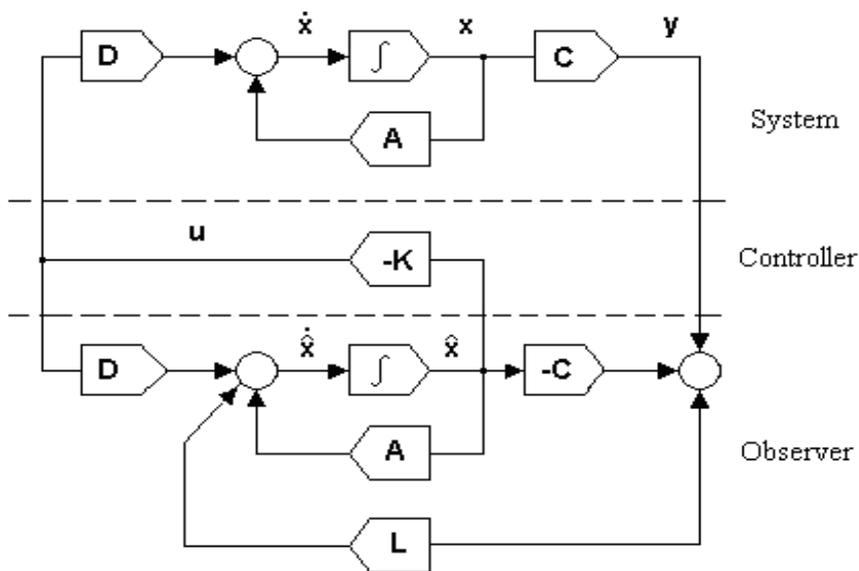


Fig. 3: System with observer-controller

8. Simulation Results

The system parameters in eq. (8) used for the simulation with MATLAB are resistance $R = 4\Omega$, $P_s = 10^6 N/m$, $m = 10^3 kg$, $L_0 = 1 Vs/A$, $P_i = 10^3 N/A$.

The weightings in eq. (10) used for the control system design are $Q_1 = 10^6$, $Q_2 = 1$, weighting $R = 10^{-4}$.

The power spectral densities in eq. (14) for the system input noise and the measurement noise are $G = 1$, $H_1 = 10^{-10}$, $H_2 = 10^{-5}$.

In Fig.4 and Fig.5 the time histories of magnet gap deviation and vertical acceleration are presented with a gap initial condition $s(0) = 2 mm$ for direct state feedback and the use of an observer-controller.

9. Conclusions

For a MAGLEV vehicle the equations of motion have been derived with LAGRANGE-Function and written in state space notation. It was possible to stabilize the system with the aid of Linear Quadratic Control and an observer (KALMAN-Filter). The time histories of magnet gap deviation and vertical acceleration are presented with a gap initial condition. The results using an observer-controller compared with the results of a direct state feedback show that the MAGLEV vehicle can be also successfully stabilized with a state estimation carried out by an observer.

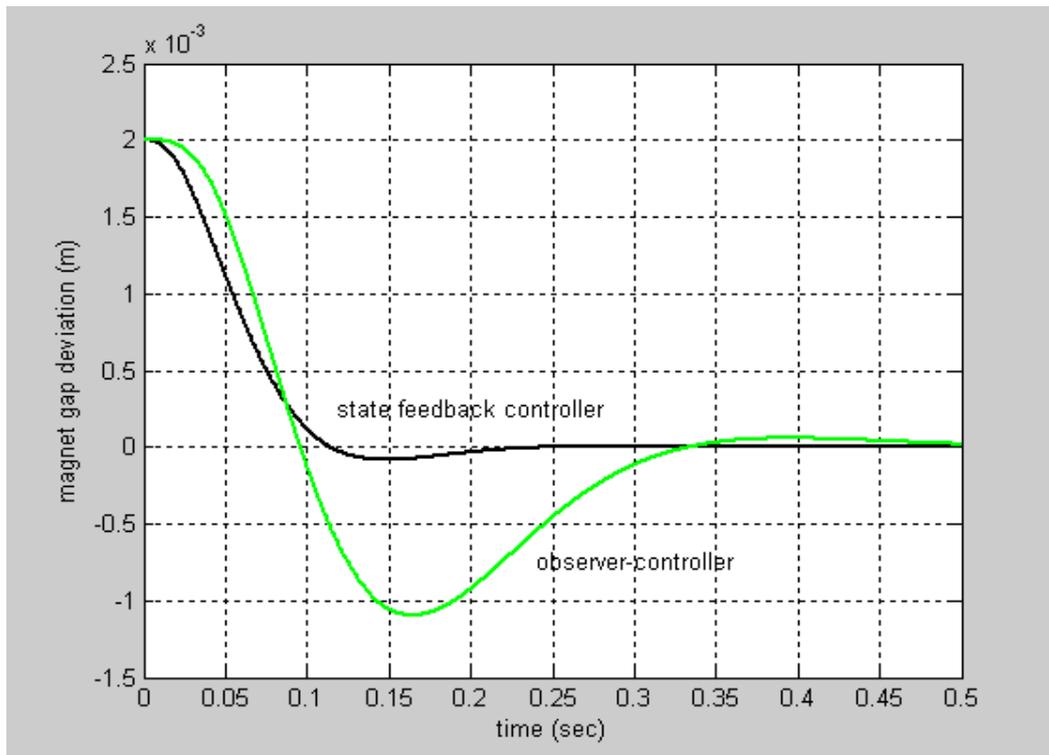


Fig. 4: Time histories of magnet gap with a gap initial condition $s(0)=2\text{mm}$

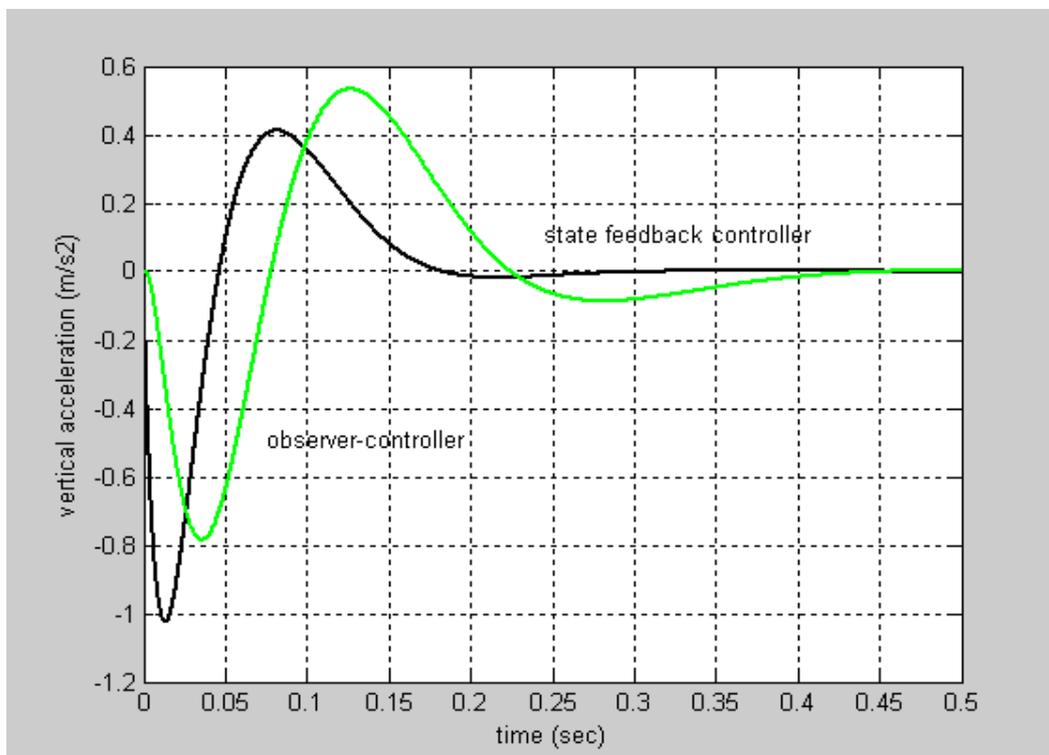


Fig. 5: Time histories of acceleration with a gap initial condition $s(0)=2\text{mm}$

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